

Exercise 13

Find dy/dx by implicit differentiation.

$$\sqrt{x+y} = x^4 + y^4$$

Solution

Differentiate both sides with respect to x .

$$\frac{d}{dx}(\sqrt{x+y}) = \frac{d}{dx}(x^4 + y^4)$$

$$\frac{1}{2}(x+y)^{-1/2} \cdot \frac{d}{dx}(x+y) = \frac{d}{dx}(x^4) + \frac{d}{dx}(y^4)$$

$$\frac{1}{2}(x+y)^{-1/2} \cdot (1+y') = 4x^3 + 4y^3 \cdot \frac{d}{dx}(y)$$

$$\frac{1+y'}{2\sqrt{x+y}} = 4x^3 + 4y^3y'$$

$$1+y' = 8x^3\sqrt{x+y} + 8y^3y'\sqrt{x+y}$$

Solve for y' .

$$y'(1 - 8y^3\sqrt{x+y}) = 8x^3\sqrt{x+y} - 1$$

$$y' = \frac{8x^3\sqrt{x+y} - 1}{1 - 8y^3\sqrt{x+y}}$$